Ants Can Colour Graphs

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Assignment Type Problems

Problems involving the allocation of limited resources to jobs.

In this particular case, the graph colouring problem

The aim here is to minimise the number of colours used to colour a graph
Real-World Applications

- Many situations where limited resources must be shared between tasks; for example:
  - Register allocation in computers
  - Machine allocation in production tasks
  - Radio frequency allocation in telecommunications
Previous Approaches

- **Constructive Methods**
  - Start with an empty solution and build up from there
  - 2 decisions at each step:
    - Which vertex to choose
    - What resource to assign

- **Sequential Methods**
  - Start at a feasible solution and move step-by-step to a new solution (not analysed in this paper)

- **Evolutionary Methods**
  - Manipulate a set of solutions located in different areas of the problem space (e.g. ant colony)
Constructive Methods

- **Static Orders** (determined at the start)
  - Vertices ordered randomly
  - Largest First
  - Smallest Last
- **Dynamic Orders** (dependent on previous colour selections)
  - DSATUR (choose vertex based on saturation)
  - Recursive Largest First
Ant-Based System Model

- ANTCOL, a hybrid constructive/evolutionary model
- Keep a record of best value so far
- Stop after a certain number of cycles
- For every cycle:
  - For every ant:
    - Colour the graph via some constructive method
    - If the colouring is better than any so far, record the result
  - Every ant contributes to the pheromone trail, but only after all ants have had a turn
- The constructive methods used in this study are RLF and DSATUR
ANT_RLF

- 6 sub-variants
  - \( \sum = 1 \) or \( 2 \) (strategy for first vertex selection)
    - 1: choose the vertex with highest number of uncoloured adjacent available vertices
    - 2: choose randomly
  - \( \Omega = 1, 2 \) or \( 3 \) (strategy for calculating vertex desirability)
    - 1: vertex probability determined by the number of adjacent vertices which cannot be in the stable set being built
    - 2: vertex probability determined by the total number of uncoloured vertices minus the number of number of uncoloured adjacent vertices
    - 3: vertex probability determined by the number of adjacent vertices which are either uncoloured or cannot be part of the stable set being built
ANT_DSATUR

- Vertex probability determined by the number of neighbour vertices already coloured
- $\Psi = 1$: choose the smallest available colour, vertex choice probability affected by the current pheromone trail
- $\Psi = 2$: vertex choice unaffected by current pheromone trail; choose colour based on current trail
Other variations

- Various factors were tested independently of each other:
  - The trail ($\beta$) and vertex desirability ($\alpha$) factors were varied in the set $\{0,1,2,3,4\}$ to discover which values were best.
  - Trail evaporation was varied between $[0..1.0]$, incrementing by 0.1.
  - Number of ants was varied between $[20, 200]$, incrementing by 20. Some tests compared ant colonies of sizes 100 and 300.
Discussion of Results

- Up to this study, the best ANTCOL result for the problem $G_{100,0.5}$ was 15.30 colours (on average), this study.
- This study achieved a result of 15.05 on average
- The best known result for that particular set of graphs was 14.95
Results: Decay probability

- **RLF**
  - Worst results when $q > 0.8$
  - Otherwise results ranged between 15.4 and 15.7 colours

- **DSATUR**
  - Worst results at $q = 1$
  - Otherwise curve decreases from around 16 to 15.6 colours
Results: Varying $\beta$

- B was set to either 2 or 4
- RLF
  - Vertex desirability strategy 1
    - Better performance for $\beta = 4$
  - Vertex desirability strategy 2
    - Better performance for $\beta = 2$
- DSATUR
  - Better performance for $\beta = 2$
Colony Size

- **RLF**
  - Very little variability between colony sizes

- **DSATUR**
  - Results poor for colonies with less than 50 ants
  - Results continue improving as ants are added
Results: Number of Cycles

- **RLF**
  - Larger ant colonies resulted in more colours
  - Shape of curve unaffected by number of ants
  - Near-optimal results reached at around 50 cycles

- **DSATUR**
  - Larger ant colonies resulted in less colours
  - Near optimal results reached at around 100 cycles

- **In both cases:**
  - Graph size was
  - Larger ant colony sizes meant a longer time before optimal results were reached
Critical Evaluation of the Approach

- The approach involved adapting an existing heuristic and testing the adaptations to see if they gave better results than previously seen.
- No better results were obtained than those currently existing.
- Statistics presented were limited (no standard deviation or other non-elementary statistics).
This paper added the following information, although it doesn’t explicitly state it as such:

- Overall DSATUR produces better results than RLF
- RLF takes fewer cycles than DSATUR
- The effects of varying different parameters on the effectiveness and efficiency of ANTCOL when using RLF and DSATUR
The End

- Thank you for attending my presentation
- Questions and Discussion